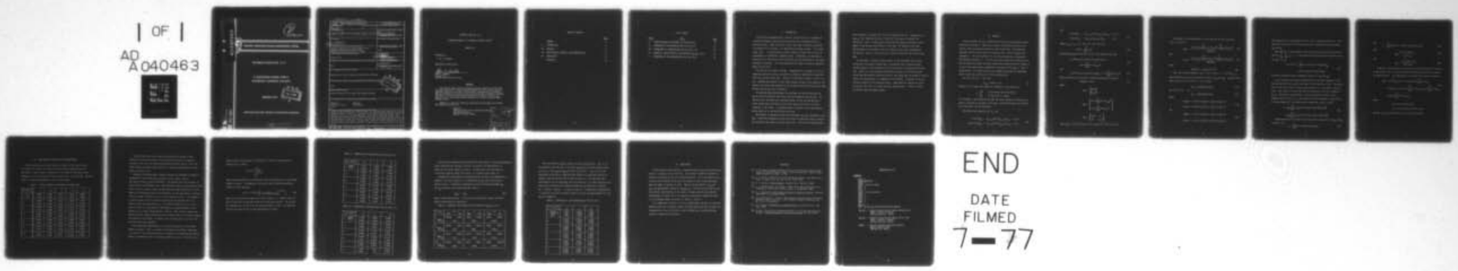
AD-A040 463 DEFENSE COMMUNICATIONS ENGINEERING CENTER RESTON VA F/G 12/2
A QUEUEING MODEL FOR A TECHNICAL CONTROL FACILITY.(U)
MAR 77 M J FISCHER
UNCLASSIFIED TN-5-77 NL

1 OF 1
AD A040463



END
DATE FILMED
7-77

2
NW TN 5-77

DEFENSE COMMUNICATIONS ENGINEERING CENTER

TECHNICAL NOTE NO. 5-77

A QUEUEING MODEL FOR A TECHNICAL CONTROL FACILITY

MARCH 1977

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED

UNCLASSIFIED R720

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

14

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
TN-5-77

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
6 A QUEUEING MODEL FOR A TECHNICAL CONTROL FACILITY.

5. TYPE OF REPORT & PERIOD COVERED
9 Technical Note,

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
10 M. J. Fischer

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
407519
Defense Communications Engineering Center
~~Systems Engineering Division~~ R700
1860 Wiehle Avenue, Reston, VA 22090

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
N/A

11. CONTROLLING OFFICE NAME AND ADDRESS
(same as 9)

12. REPORT DATE
11 Mar[illegible]977

13. NUMBER OF PAGES
12 20p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
N/A

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
N/A

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distribution unlimited.

DDC
JUN 13 1977
C

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
N/A

18. SUPPLEMENTARY NOTES
Review relevance five years from submission date.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Queueing	Analysis
Technical Control	Multiserver
Model	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper we develop a queueing model for one possible operating rule for a technical control facility. As suggested in reference [1] a military technical control facility can be considered as a multiserver queueing system. One possible operating rule, the situation where one and only one of the servers may periodically leave the facility, has not been considered previously in the queueing theory literature. In this paper, we give an exact analysis, as well as presenting several approximations, to the steady state behavior of the system.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

407519 UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

TECHNICAL NOTE NO. 5-77

A QUEUEING MODEL FOR A TECHNICAL CONTROL FACILITY

MARCH 1977

Prepared by:

- M. J. Fischer

Approved for Publication:

GINO J. COVIELLO
Acting Chief
Systems Engineering Division

FOREWORD

The Defense Communications Engineering Center (DCEC) Technical Notes (TN's) are published to inform interested members of the defense community regarding technical activities of the Center, completed and in progress. They are intended to stimulate thinking and encourage information exchange; but they do not represent an approved position or policy of DCEC, and should not be used as authoritative guidance for related planning and/or further action.

Comments or technical inquiries concerning this document are welcome, and should be directed to:

Director
Defense Communications Engineering Center
1860 Wiehle Avenue
Reston, Virginia 22090

TABLE OF CONTENTS

LIST OF TABLES

I. INTRODUCTION

A military telecommunications technical control facility is composed of a group of technical control operators who have the responsibility of restoring circuits. When one of the circuits goes down a technical controller is assigned to it to ensure it is repaired and brought up again. In a recent study [1], it was pointed out that a technical control facility can be considered as a multiserver queueing system. Furthermore, one possible operating rule for the facility had not previously been considered in the queueing theory literature. It is the purpose of this paper to present a queueing model for this rule.

The rule can be stated as follows: Independent of the number of circuits requiring corrective action, one technical controller periodically leaves the facility to perform other work. After he is finished he returns to the facility. Although the analysis of this paper assumes only one of the technical controllers may leave, the extension to the situations when more than one may leave follows via the same sort of analysis.

In queueing theory terminology, the customers are the circuits and the servers are the technical controllers who are assigned to the facility. For the particular operating rule considered above, we will be considering an s server system where periodically one server leaves the facility to return sometime in the future. In queueing theory literature, we are considering a system where one of the servers may break down.

The analysis of queueing systems with breakdowns has been considered in the past. Gaver [2] introduced and used the notion of completion times to analyze the case where the number of servers equals one. Several other related papers have appeared, e.g. Jaiswal [3]. We will not mention them all, although one, reference [4], should be noted as it was the first paper to consider the multiserver case. In that paper, Metrany and Avi-Itzhak considered a system where all the servers may go away or break down The operating rule under consideration in this paper is sort of a hybrid of the rules considered in [2] and [4]. That is, only one of the s servers leaves the facility or breaks down.

In this paper, we give an exact analysis of the case where the circuits (customers) are assumed to break down in accordance with a Poisson process and the length of time required to fix the circuit has an exponential distribution. There are s (≥ 1) technical controllers (servers) in the facility and one of them periodically leaves the facility. We assume that the length of time he leaves and stays, each has an exponential distribution. In section II we give an exact analysis of this queueing system. Some numerical examples are given in section III, as well as several possible approximations. Finally, section IV contains some concluding remarks.

II. ANALYSIS

In this section, we give a mathematical analysis of the queueing system described in section I. There are s servers and periodically one of the servers leaves the system to return at some future time. We assume that customers arrive to the system in accordance with a Poisson process with parameter λ. The service time of an arriving customer has an exponential distribution with mean μ^{-1}. For the server who periodically leaves the system, we assume the random variables representing length of time he remains in or out of the system have an exponential distribution with means α^{-1} and β^{-1} respectively. All random variables are assumed to be mutually independent and we assume the system has an infinite waiting room.

Let, for n=0,1,2,..., and i=0,1

$$P_{n,i} = Pr\{Q=n,\ Y=i\}, \qquad (1)$$

where Q is the steady state number of customers in the system and

$$Y = \begin{cases} 0 & \text{if the server away from system} \\ 1 & \text{if the server in system.} \end{cases}$$

If a customer is receiving service when the server decides to leave, the customer is returned to the head of the queue. Waiting customers are serviced on a first-come, first-served basis.

The steady state equations for $P_{n,i}$ are

$$\begin{aligned} (\lambda+n\mu+\beta)P_{n,0} &= \lambda P_{n-1,0}+(n+1)\mu P_{n+1,0}+\alpha P_{n,1}\ ; \quad n\leq s-2 \\ (\lambda+(s-1)\mu+\beta)P_{n,0} &= \lambda P_{n-1,0}+(s-1)\mu P_{n+1,0}+\alpha P_{n,1}\ ; \quad n\geq s-1 \end{aligned} \qquad (2)$$

and

$$
\begin{aligned}
(\lambda+n\mu+\alpha)P_{n,1} &= \lambda P_{n-1,1}+(n+1)\mu P_{n+1,1}+\beta P_{n,0} \quad ; \quad n\leq s-1 \\
(\lambda+s\mu+\alpha)P_{n,1} &= \lambda P_{n-1,1}+s\mu P_{n+1,1}+\beta P_{n,0} \quad ; \quad n\geq s .
\end{aligned} \tag{3}
$$

where $P_{-1,0} = P_{-1,1} \equiv 0$. Let, for $i=0,1$ and $|z|\leq 1$,

$$P_i(z) = \sum_{n=0}^{\infty} P_{n,i} z^n . \tag{4}$$

Multiplying equations (2) and (3) by z^n and combining one gets

$$
\begin{aligned}
&(-\lambda z^2+(\beta+\lambda+(s-1)\mu)z-(s-1)\mu)P_0(z)-\alpha z P_1(z) \\
&=\mu(1-z)\sum_{n=0}^{s-1}(n+1-s)P_{n,0}z^n
\end{aligned} \tag{5}
$$

and

$$(-\lambda z^2+(\alpha+\lambda+s\mu)z-s\mu)P_1(z)-\beta z P_0(z) = \mu(1-z)\sum_{n=0}^{s-1}(n-s)P_{n,1}z^n . \tag{6}$$

These equations can be written in the following matrix equation:

$$A(z)P(z) = B(z) \tag{7}$$

where

$$P(z) = \begin{bmatrix} P_0(z) \\ P_1(z) \end{bmatrix}$$

$$B(z) = \begin{bmatrix} \mu(1-z)\sum_{n=0}^{s-1}(n+1-s)P_{n,0}z^n \\ \\ \mu(1-z)\sum_{n=0}^{s-1}(n-s)P_{n,1}z^n \end{bmatrix}$$

$$A(z) = \begin{bmatrix} a_0(z) & -\alpha z \\ -\beta z & a_1(z) \end{bmatrix}$$

with $a_0(z)=-\lambda z^2+(\beta+\lambda+(s-1)\mu)z-(s-1)\mu$ and $a_1(z)=-\lambda z^2+(\alpha+\lambda+s\mu)z-s\mu$.

If $\det(A(z))$ is the determinant of $A(z)$ then the solution for $P_i(z)$, $i=0,1$, is given by

$$P_0(z) = \frac{\mu(1-z)\{a_1(z)\sum_{n=0}^{s-1}(n+1-s)P_{n,0}z^n+\alpha z\sum_{n=0}^{s-1}(n-s)P_{n,1}z^n\}}{\det(A(z))} \tag{8}$$

and

$$P_1(z) = \frac{\mu(1-z)\{a_0(z)\sum_{n=0}^{s-1}(n-s)P_{n,1}z^n+\beta z\sum_{n=0}^{s-1}(n+1-s)P_{n,0}z^n\}}{\det(A(z))} \tag{9}$$

where

$$\det(A(z)) = a_0(z)a_1(z)-\alpha\beta z^2 . \tag{10}$$

Thus, the solution depends on $P_{n,i}$ ($i=0,1$; $n=0,1,2,\ldots,s-2$) and $P_{s-1,1}$. These probabilities may be expressed in terms of $P_{0,0}$ and $P_{0,1}$. Consider equations (2) and (3), for $n=0,1,2,\ldots,s-2$ we have

$$P_{n,0} = C_0(n)P_{00}+C_1(n)P_{01} \tag{11}$$

and for $n=0,1,2,\ldots,s-1$

$$P_{n,1} = D_0(n)P_{00}+D_1(n)P_{01} \tag{12}$$

where

$$\begin{aligned} n\mu C_0(n) &= (\lambda+\beta+(n-1)\mu)C_0(n-1)-\lambda C_0(n-2)-\alpha D_0(n-1) \\ n\mu C_1(n) &= (\lambda+\beta+(n-1)\mu)C_1(n-1)-\lambda C_1(n-2)-\alpha D_1(n-1) \end{aligned} \tag{13}$$

and

$$\begin{aligned} n\mu D_0(n) &= (\lambda+\alpha+(n-1)\mu)D_0(n-1)-\lambda D_0(n-2)-\beta C_0(n-1) \\ n\mu D_1(n) &= (\lambda+\alpha+(n-1)\mu)D_1(n-1)-\lambda D_1(n-2)-\beta C_1(n-1) \end{aligned} \tag{14}$$

with $C_0(0)=D_1(0)\equiv 1$, $C_1(0)=D_0(0)\equiv 0$ and for $i=0,1$ $C_i(n)=D_i(n)\equiv 0$ when $n<0$. This shows that the solution to the problem rests on determining the two unknowns P_{00} and P_{01}.

One equation in these unknowns can be found by the normalizing condition; that is, $\sum_{n=0}^{\infty} \sum_{i=0}^{1} P_{n,i} = 1$. From equations (8) and (9), using L'Hospital's rule, one gets ($\rho=\lambda/\mu$)

$$s-\rho-{}^{\alpha}/(\alpha+\beta) = \sum_{n=0}^{s-1}\{[(s-1-n)C_0(n)+(s-n)D_0(n)]P_{0,0} + [(s-1-n)C_1(n)+(s-n)D_1(n)]P_{0,1}\}. \tag{15}$$

We need to generate another independent equation in $P_{0,0}$ and $P_{0,1}$.

Let us consider the equation $\det(A(z))=0$ by using Sturm sequences, [5]; one can show there are exactly two roots of this equation in [0,1]. Obviously, one of the roots is equal to one, and note that if $s=1$, the other is equal to zero. Thus, there is exactly one root of $\det(A(z))=0$ in [0,1); we denote it by z_0. Returning to equations (8) or (9), the numerator of each of these equations must vanish at z_0. This gives us two more equations in $P_{0,0}$ and $P_{0,1}$. A little algebra will show that they are equivalent, and so we have

$$0 = P_{0,0}\{\sum_{n=0}^{s-1}[a_1(z_0)(n+1-s)C_0(n)-\alpha z_0(s-n)D_0(n)]z_0^n\} + P_{0,1}\{\sum_{n=0}^{s-1}[(a_1(z_0)(n+1-s)C_1(n)-\alpha z_0(s-n)D_1(n)]z_0^n\}. \tag{16}$$

From equations (15) and (16) we can now give an expression for $P_{0,0}$ and $P_{0,1}$. For $i=0,1$, let

$$\gamma_i = \sum_{n=0}^{s-1}[(s-1-n)C_i(n)+(s-n)D_i(n)] \tag{17}$$

and

$$\eta_i = \sum_{n=0}^{s-1} [a_1(z_0)(n+1-s)C_i(n) - \alpha z_0(s-n)D_i(n)]z_0^n \tag{18}$$

then

$$P_{0,0} = \frac{(s-\rho-\alpha/(\alpha+\beta))\eta_1}{\gamma_0\eta_1-\gamma_1\eta_0} \tag{19}$$

and

$$P_{0,1} = -\frac{(s-\rho-\alpha/(\alpha+\beta))\eta_0}{\gamma_0\eta_1-\gamma_1\eta_0} . \tag{20}$$

Although no simple expression exists for the expected number of customers in the system, $E(Q)$, one may differentiate equations (8) and (9) to obtain the desired result. For completeness, we include the resulting expression

$$\begin{aligned} E(Q) = (\mu/\Theta_1^2)\{&[\Theta_0(\alpha+\beta-\lambda+(s-1)\mu)-\Theta_1(\alpha+\beta)] \sum_{n=0}^{s-1} (n-s)P_{n,1} \\ &+ [\Theta_0(\alpha+\beta-\lambda+s\mu)-\Theta_1(\alpha+\beta)] \sum_{n=0}^{s-1} (n+1-s)P_{n,0} + \Theta_0(\alpha+\beta) \sum_{n=0}^{s-1} n(n-s)P_{n,1} \\ &+ \Theta_0(\alpha+\beta) \sum_{n=0}^{s-1} n(n+1-s)P_{n,0}\} \end{aligned} \tag{21}$$

where

$$\Theta_0 = \beta(\lambda-s\mu)+\alpha(\lambda-(s-1)\mu)$$

$$\Theta_1 = (\lambda-\beta-(s-1)\mu)(\alpha-\lambda+s\mu)+\beta\lambda+\alpha\lambda+\alpha\beta,$$

and the $P_{n,i}$'s are found from (11), (12), (19) and (20).

III. SOME NUMERICAL EXAMPLES AND APPROXIMATIONS

In this section we give some numerical examples of the results found in section II, as well as discussing several possible approximations to the system. Table I gives a comparison of the system for the same traffic intensity, $\rho=\lambda/\mu(\mu=1)$, and two different values of s, (s=5 and s=10). We note that as $\beta\to 0$ the system behaves as an s-1 server system.

TABLE I. EXPECTED NUMBER OF CUSTOMERS IN SYSTEM (E(Q))

ρ / s \ $\alpha/(\alpha+\beta)$		0	.25	.5	.75	1.0
.2	s=5	1.0010	1.0022	1.0037	1.0051	1.0068
	s=10	2.0000	2.0000	2.0000	2.0000	2.0000
.3		1.5086	1.5160	1.5247	1.5338	1.5447
		3.0005	3.0008	3.0012	3.0015	3.0020
.4		2.0398	2.0651	2.0965	2.1305	2.1739
		4.0059	4.0089	4.0121	4.0154	4.0190
.5		2.6304	2.6987	2.7881	2.8910	3.0330
		5.0361	5.0501	5.0659	5.0821	5.1006
.6		3.3540	3.5222	3.7577	4.0584	4.5283
		6.1519	6.2007	6.2579	6.3189	6.3920
.7		4.3816	4.8050	5.4721	6.5106	8.6650
		7.5174	7.6685	7.8559	8.0707	8.3473
.78		5.7302	6.7425	8.6775	13.1717	40.7593
		9.0976	9.4881	10.0111	10.6802	11.6563

From this table one can see that the system with 5 servers is more sensitive to one of the servers leaving than the system with 10 servers. Of course, this was to be expected since when one server leaves in the five server system, the total system capacity is reduced by 20%, whereas, in the other case only by 10%.

Although a straightforward, standard analysis was presented in section II development of some simple approximations to the system that do not require finding z_0 may be desirable. One approach may be to tend the desired measure of performance (e.g., expected queue length, average waiting time) for an s and s-1 server system, and then take the convex combination of these measures of performance based on the proportion of time the system is operating like an s server ($\beta/(\alpha+\beta)$) and an s-1 server system ($\alpha/(\alpha+\beta)$). This approach is not very good, since the traffic intensity may be such that for an s-1 server system, the system blows up. For example, suppose $s=5$, $\alpha=1$, $\beta=2$, $\mu=1$, and $\lambda=4$; since $\rho=4$, the measure of performance, say expected number in the system, for a 4 server system would be infinite. Thus, taking a convex combination of this number and the case for s=5 would result in an infinite number of customers in the system. Of course, this is wrong since $\rho<s-\alpha/(\alpha+\beta)$, and thus the Cesaro mean converges.

A more promising approximation is to treat the system as a non-integer number of servers. That is, consider the system to be an M/M/s' system when $s'=s-\alpha/(\alpha+\beta)$. The only problem with this approach is in computing the desired measure of performance for a non-integer number of servers. From [7] one can express most of the measures of performance in terms of the Erlang Loss Formula, $E(s,\rho)$, where

$$E(s,\rho) = \frac{\rho^s/s!}{\sum_{j=0}^{s} \rho^j/j!} .$$

Thus, the only problem is to determine Erlang's Loss Formula for a non-integer number of servers. As suggested in [8] we use the following extrapolation formula for this quantity,

$$E(s',\rho) = E([s'],\rho)\left[\frac{\rho}{[s']+1+\rho E([s'],\rho)}\right]^{s'-[s']}, \tag{22}$$

when $[x]$ is the greatest integer less than or equal to x. Tables II and III give a comparison of this approximation with the exact results. The results are presented for s=5 and s=10, and various values of α and β. We note that for the case where α=0 or β=0 the approximation is exact.

TABLE II. COMPARISON OF APPROXIMATION FOR E(Q) WITH s=5

ρ/s \ $\alpha/(\alpha+\beta)$		.25	.5	.75
.2	APPROX	1.0015	1.0025	1.0041
	EXACT	1.0022	1.0037	1.0051
.3		1.5128	1.5193	1.5292
		1.5160	1.5247	1.5338
.4		2.0565	2.0813	2.1181
		2.0651	2.0965	2.1305
.5		2.6811	2.7549	2.8649
		2.6987	2.7881	2.8910
.6		3.4907	3.6944	4.0104
		3.5222	3.7577	4.0584
.7		4.7529	5.3544	6.4231
		4.8050	5.4721	6.5106

TABLE III. COMPARISONS OF APPROXIMATION FOR E(Q) WITH s=10

ρ/s \ $\alpha/(\alpha+\beta)$		.25	.5	.75
.2	APPROX	2.0000	2.0000	2.0000
	EXACT	2.0000	2.0000	2.0000
.3		3.0007	3.0010	3.0014
		3.0008	3.0012	3.0015
.4		4.0078	4.0105	4.0141
		4.0089	4.0121	4.0154
.5		5.0464	5.0598	5.0774
		5.0501	5.0659	5.0821
.6		6.1910	6.2414	6.3067
		6.2007	6.2579	6.3189
.7		7.6480	7.8185	8.0441
		7.6685	7.8559	8.0707

Two points are immediately discernible from these tables. First, the approximation always underestimates the exact results, and second, the approximation is better for the larger number of servers. In both cases, the approximation is extremely good and seems to be worst, in a relative sense, when $\alpha=\beta$.

Although this approximation is extremely good, the only real computational problem in the exact analysis is in determining z_0, the root of $\det(A(z))=0$ inside of [0,1]. A numerical investigation of the relationship between $P_{0,0}$ and $P_{0,1}$ revealed a very interesting fact; that is,

$$\beta P_{0,0} \doteq \alpha P_{0,1} \tag{23}$$

where $\doteq$ means approximately. Table IV gives some typical results that were used in generating this observation

TABLE IV. NUMERICAL INVESTIGATION OF THE EQUATION $\beta P_{00} \doteq \alpha P_{01}$ (s=5)

ρ \ α/β	1/3 $P_{0,0}$	1/3 $P_{0,1}$	1 $P_{0,0}$	1 $P_{0,1}$	3/1 $P_{0,0}$	3/1 $P_{0,1}$
1	.091928	.27582	.18379	.18380	.27561	.09187
$P_{0,0}/P_{0,1}$	.3333		.9999		2.9999	
2	.03338	.10017	.06626	.06636	.09872	.03292
$P_{0,0}/P_{0,1}$	.3333		.9986		2.9992	
3	.01125	.03380	.02146	.02160	.03056	.01020
$P_{0,0}/P_{0,1}$	.3329		.9933		2.9957	
3.9	.00326	.00981	.00518	.00527	.00534	.0017877
$P_{0,0}/P_{0,1}$	.3322		.9825		2.9878	

This relationship suggests another possible approximation: that is, to use equations (15) and (23) to solve for $P_{0,0}$ and $P_{0,1}$ directly without having to find z_0. Once $P_{0,0}$ and $P_{0,1}$ are found, $P_{n,i}$ ($n=1,2,\ldots$ and $i=0,1$) can be determined recursively, or expected value measures of performance can be obtained as was done in the case of $E(Q)$ in section II. We note that this approximation has two advantages over the other one: it is more accurate, and it can be used to generate the complete probability distribution if required. The greater accuracy is shown in Table V. In that table, approximation I is the one using the non-integer number of servers and approximation II, the one just suggested.

TABLE V. COMPARISON OF TWO APPROXIMATIONS FOR E(Q) (s=5)

ρ/s \ $\alpha/(\alpha+\beta)$		.25	.5	.75
.2	APPROX. I	1.0015	1.0025	1.0041
	APPROX. II	1.0021	1.0036	1.0050
	EXACT	1.0022	1.0037	1.0051
.3		1.5128	1.5193	1.5292
		1.5157	1.5243	1.5335
		1.5160	1.5247	1.5338
.4		2.0565	2.0813	2.1181
		2.0644	2.0956	2.1295
		2.0651	2.0965	2.1305
.5		2.6811	2.7549	2.8649
		2.6973	2.7860	2.8890
		2.6987	2.7881	2.8910
.6		3.4907	3.6944	4.0104
		3.5198	3.7540	4.0504
		3.5222	3.7577	4.0548
.7		4.7529	5.3544	6.4231
		4.8015	5.4664	6.5050
		4.8050	5.4721	6.5106

IV. CONCLUSIONS

In this paper we have presented a queueing model of one possible operating rule of a techncial control facility. The numerical examples considered in section III pointed out serveral interesting facts. First, the effect of one of the servers leaving the facility for periods of time is more pronounced when the number of servers is small. Second, the two unknowns, $P_{0,0}$ and $P_{0,1}$, are approximately related as $\beta P_{0,0} \doteq P_{0,1}\alpha$. This fact can be used to give one possible approximation to the system. Another possible approximation, investigated in section III, is to consider an equivalent system when there is a non-integer number of servers, s' with $s' = -\alpha/(\alpha+\beta)$.

It is hoped that the exact, as well as approximate, analysis of a possible operating rule for a technical control facility may provide some insights into the operation of such a facility as well as being used in performing some trade-off studies for the system.

REFERENCES

[1] F. H. Sawyer, "Manpower Analyses Concerning DCS Technical Control Operations in Europe (Minature Study)," Internal Memorandum, Defense Communications Engineering Center, (1976)

[2] D. P. Gaver, "A Waiting Line with Interrupted Service, Including Priorities," J. Royal Stat. Soc. Ser. B, pp 73-90 (1962)

[3] N. K. Jaiswal, Priority Queues, Academic Press, New York (1968)

[4] I. L. Metrany and B. Avi-Itzhak, "A Many-Server Queue with Service Interruptions," Operations Research, 16 No. 3, pp 628-638 (1968)

[5] A. S. Housholder, The Theory of Matrices in Numerical Analysis, Blaisdell Publishing Co., New York (1964)

[6] U. N. Bhat and M. J. Fischer, "Multichannel Queueing Systems with Heterogeneous Classes of Arrivals," Naval Research Logistics Quarterly, 23, No. 2, pp 271-282(1976)

[7] R. B. Cooper, Introduction to Queueing Theory, The Macmillan Co., New York (1972)

[8] S. Katz, "Statistical Performance Analysis of a Switched Communications Network," Fifth International Teletraffic Congress, New York (1967)

DISTRIBUTION LIST

<u>STANDARD:</u>

R100 - 2
R102/R103/R103R - 1
R102M - 1
R102T - 9 (8 for stock)
R104 - 1
R110 - 1
R123 - 1
R124A - 1 (for Archives)
R200 - 1
R300 - 1
R500 - 1
R700 - 1
R800 - 1
931 - 1
NCS-TS - 1
205 - 20 (Unclassified/Unlimited Distribution)

DCA-EUR - 1 (Defense Communications Agency European Area
ATTN: Technical Director
APO San Francisco 96515)

DCA-PAC - 1 (Defense Communications Agency Pacific Area
ATTN: Technical Director
APO San Francisco 96515)

USDCFO - 1 (Unclassified/Unlimited Distribution)
(Chief, USDCFO/US NATO
APO New York 09667)